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Updated Study of a Precision Measurement of the W Mass from a Threshold Scan Using Polarized e^- and e^+ at ILC

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An updated study of measuring the W mass from a polarized threshold scan at ILC is presented with an emphasis on evaluating scan strategies that control experimental systematics. Highly longitudinally polarized beams of electrons and positrons such as are feasible at ILC offer significant advantages in terms of statistical power and in-situ control of background. Eventual experimental precision of around 2 MeV can be envisaged from this technique. Further work on both the accelerator design and theoretical uncertainties will likely be needed to take full advantage of this opportunity.

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Introduction

A future high energy e^+e^- collider is recognized as essential for a precision study of the Higgs and the top quark [1]. It can also be a very powerful tool for advancing measurements of precision electroweak observables [1–4]. One of those observables of considerable importance is the W mass. Measurements from LEP2 and the Tevatron have led to a current precision of 15 MeV [5]. Further improvements from long existing hadron collider data-sets at the Tevatron and LHC are possible, but given the predominant systematic uncertainties will constitute major experimental and phenomenological tours de force if and when they are realized.

The three most promising approaches to measuring the W mass at an e^+e^- collider are:

Polarized Threshold Scan Measurement of the W^+W^- cross-section near threshold with longitudinally polarized beams.

Constrained Reconstruction Kinematically-constrained reconstruction of W^+W^- using constraints from four-momentum conservation and optionally mass-equality as was done at LEP2.

Hadronic Mass Direct measurement of the hadronic mass. This can be applied particularly to single-W events decaying hadronically or to the hadronic system in semi-leptonic W^+W^- events.

Methods for measuring the W mass in e^+e^- colliders were explored extensively in the LEP era, see [6, 7] and references therein.

The International Linear Collider (ILC) is designed to reach $\sqrt{s} \approx 500$ GeV with polarized beams and high luminosity and can be upgraded to $\sqrt{s} \approx 1$ TeV. With the envisaged accelerator parameters and ILC operating scenarios [8], there is a great potential for much improved measurements of the W mass. With the example operating scenario H-20 from [8], one can envisage data-sets totaling up to 6200 fb^{-1} at center-of-mass energies of between 250 and 500 GeV. This is an energy regime where the W mass can be measured using the constrained reconstruction and hadronic mass techniques. The eventual uncertainty would almost certainly be limited by experimental systematics. Previous rough estimates by the author in [4] suggest experimental systematics on the 3-4 MeV scale which would dominate the statistical uncertainties. Nevertheless given the opportunity to improve on the measurement of the W mass using data collected synergistically with the main ILC physics program this is an area where further detailed study would be very welcome.

The main subject of this contribution is an update of a previous study on the measurement of the W mass using a polarized threshold scan [9, 10]. The study has evolved taking into account additional systematic effects. The updates include the use of beam parameters consistent with the ILC TDR design and experimental

performance appropriate to the envisaged ILC detectors. The previous study was started in 1999 and had very conservatively assumed experimental characteristics similar to the LEP detectors.

Such a measurement would necessarily entail significant allocation of running time to data-taking near $\sqrt{s} = 161$ GeV where the cross-section is most sensitive to M_W . Low statistics measurements at a single center-of-mass energy with unpolarized beams were done in 1996 by the LEP experiments [11–14] and were reviewed in [15,16]. The dependence of the cross-section on center-of-mass energy is illustrated in Figure 1. There are two primary experimental issues at the heart of interpreting a high statistics threshold scan as a measurement of the W mass. Firstly, one needs excellent control of the **absolute center-of-mass energy**, and secondly one needs to be able to control the **background**. These are discussed further in the next section.

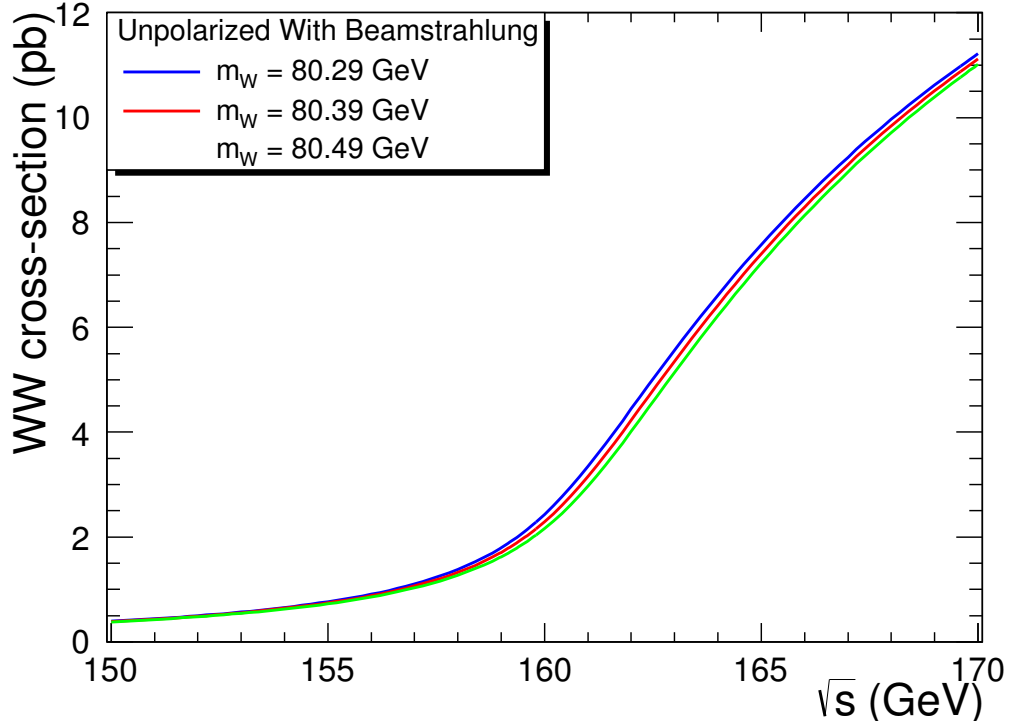


Figure 1: The unpolarized CC03 cross-section for WW production vs center-of-mass energy. The cross-section is evaluated with GENTLE2.0 including ILC beamstrahlung.

Strategy for Primary Experimental Systematics

It has been shown recently [17] that $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ events can be used to make an *in situ* measurement of the average center-of-mass energy with high statistical precision* based simply on momentum measurements of the muons as first discussed in [18]. Under the assumption that the recoil mass to the measured muons is zero, an estimate of the center-of-mass energy of such events can be formed simply from the measured momenta of the two muons:

$$\sqrt{s_P} = E_1 + E_2 + |\vec{p}_1 + \vec{p}_2| = \sqrt{p_1^2 + m_\mu^2} + \sqrt{p_2^2 + m_\mu^2} + |\vec{p}_1 + \vec{p}_2|$$

The distribution of this variable can then be used to deduce relevant parameters including those related to the average absolute center-of-mass energy. Given very good control of the tracking detector absolute momentum scale[†], it is envisaged that knowledge of the absolute center-of-mass energy at the 10 ppm level can be targeted, corresponding to a contribution to the W mass uncertainty at threshold of 0.8 MeV.

Our strategy for controlling the background is to measure the background *in situ* using polarized beams. Polarized electrons and **polarized positrons** make it feasible to measure the background and simultaneously measure the polarization (also *in situ*). Having good control of the background is critical. Assuming a background level of 250 fb, a 10% uncertainty on the background amounts to an uncertainty of 12 MeV on M_W for an unpolarized scan. With highly polarized beams, and a 100 fb⁻¹ polarized scan, the background level can be measured to about 4 fb.

In addition to the need for exclusive running near WW threshold, the polarized threshold scan is most attractive if the beams are highly polarized, and if sufficient data-sets are collected to pin-down and to monitor the tracker momentum scale which affects the determination of the absolute center-of-mass energy. At present the most obvious way to guarantee the latter, is to make sure that the accelerator can also run effectively at the Z pole yielding high statistics calibration data. In these three aspects there is a need for the ILC accelerator design to retain compatibility with good performance at relatively low energy.

*Previous studies had assumed that the absolute center-of-mass energy would be determined using the angle technique in $Z\gamma$ events (with $Z \rightarrow \mu^+\mu^-$) - a technique that suffers from relatively poor event-by-event statistical precision given the Z width

[†]Assumed known to 10 ppm through momentum-scale calibrations using at least 12,000 energetic $J/\psi \rightarrow \mu^+\mu^-$ events as outlined in [19]

Polarized Beams

The cross-section dependence on the longitudinal polarization of the electron and positron beams is given by [20]

$$\sigma(P_{e^-}, P_{e^+}) = \frac{1}{4} \{ (1 - P_{e^-})(1 + P_{e^+})\sigma_{LR} + (1 + P_{e^-})(1 - P_{e^+})\sigma_{RL} + (1 - P_{e^-})(1 - P_{e^+})\sigma_{LL} + (1 + P_{e^-})(1 + P_{e^+})\sigma_{RR} \}$$

where σ_k ($k = LR, RL, LL$ and RR) are the fully polarized cross-sections. In cases where the LL and RR cross-sections are zero, the resulting cross-section simplifies to

$$\sigma(P_{e^-}, P_{e^+}) = \frac{1}{4} \{ (1 - P_{e^-})(1 + P_{e^+})\sigma_{LR} + (1 + P_{e^-})(1 - P_{e^+})\sigma_{RL} \}$$

which can be rewritten as

$$\sigma(P_{e^-}, P_{e^+}) = \sigma_u \{ (1 - P_{e^-} P_{e^+}) - (P_{e^-} - P_{e^+})A_{LR} \} \quad (1)$$

where σ_u is the unpolarized cross-section, $(\sigma_{LR} + \sigma_{RL})/4$, and A_{LR} is the left-right asymmetry defined as

$$A_{LR} = (\sigma_{LR} - \sigma_{RL})/(\sigma_{LR} + \sigma_{RL})$$

Equation 1 is appropriate for Z production. It is also appropriate for the doubly-resonant CC03 WW production diagrams, where especially close to threshold, A_{LR} , is close to maximal[‡] given the dominance of the t-channel ν_e exchange diagram. For many of the processes, that play the role of a background to WW production, notably, $e^+e^- \rightarrow q\bar{q}gg$, it is also highly appropriate. Note that this is not appropriate for processes such as single W production which is expected to contribute to the background in the $q\bar{q}e\nu$ channel (the LL and RR cross-sections are non-zero in this case).

With the WW asymmetry of around 0.99, and background asymmetries ranging from 0.15 to 0.48 (depending on channel), it is feasible to use the polarization of the beams to preferentially enhance the signal cross-section, and therefore the statistics for the measurement. Conversely, it is possible to use the polarization to essentially turn-off the signal process, and measure events that are much enriched in background.

At ILC, it is expected that the beams can be highly polarized, and the spin can be flipped with high frequency (certainly pulse-to-pulse for electrons). Electron polarization is straightforward. Electron polarizations of 80% are in the baseline, and values as high as 90% can be targeted. The baseline ILC design has polarized

[‡]The estimated value for WW is around 0.99. Checks with Wopper [21] and RacoonWW [22] gave values of 0.992 and 0.988 near threshold

positron beams at beam energies exceeding 125 GeV with a polarization level of 30%, and there are studies and prospects for positron polarization levels as high as 60%. Fully worked out designs for high instantaneous luminosity and positron polarization at low energy are under study. The relative polarization will be monitored with polarimeters.

Experimentally, we expect to have the freedom to choose appropriate fractions of the delivered luminosity in the various polarization configurations. We can imagine collisions with positive, negative and zero polarization for each beam, resulting in a total of 9 different “helicity configurations”, namely $(-+, +-, --, ++, 00, 0+, 0-, +0, -0)$.

Example Scan

The basic method is to do a counting experiment where the experimental observables are the number of selected candidate events, N_{ijk} , consistent with WW production in the different decay channels (index i), observed at each center-of-mass energy (index j) and for each helicity configuration of the polarized electron and positron beams (index k). The data are sub-divided into the 3 major decay channels ($q\bar{q}q\bar{q}$, $q\bar{q}\ell\nu$, $\ell\nu\ell\nu$). In addition to the selected candidate events, we also measure the number of Z-like $e^+e^- \rightarrow f\bar{f}(\gamma)$ events produced for each center-of-mass energy and helicity configuration, N_{jk}^Z , as a means of measuring the polarization *in situ*. An example simulated data-set corresponding to a 6-point ILC scan with 4 helicity configurations, and correspondingly 96 event counts, is displayed in Table 1.

The observed event counts in the various channels are fitted to the expected event counts from signal plus background events in a model with fit parameters that account for the theoretical model parameters and relevant systematic effects. The fits are done using a Poisson likelihood. The fit parameters are given in Table 2.

Scan Details

The shape of the cross-section depends on m_W and Γ_W . Within the Standard Model, Γ_W , is essentially a function of m_W and α_S .

$$\Gamma_W \sim M_W^3 \left(1 + \frac{2\alpha_S(M_W^2)}{3\pi} \right) \quad (2)$$

The theoretical form of the WW CC03 cross-section is evaluated using GENTLE2.0/4fan [23]. The GENTLE predictions are convolved with the expected ILC beamstrahlung using CIRCE1 [24]. The beamstrahlung spectrum was evaluated using Guinea-Pig [25] by γ scaling of the ILC TDR accelerator parameters [26] at

\sqrt{s} (GeV)	L (fb ⁻¹)	f	$\lambda_{e-}\lambda_{e+}$	N_{ll}	N_{lh}	N_{hh}	N_{RR}
160.6	4.348	0.7789	−+	2752	11279	12321	926968
		0.1704	+-	20	67	158	139932
		0.0254	++	2	19	27	6661
		0.0254	--	21	100	102	8455
161.2	21.739	0.7789	−+	16096	67610	73538	4635245
		0.1704	+-	98	354	820	697141
		0.0254	++	37	134	130	33202
		0.0254	--	145	574	622	42832
161.4	21.739	0.7789	−+	17334	72012	77991	4639495
		0.1704	+-	100	376	770	697459
		0.0254	++	28	104	133	33556
		0.0254	--	135	553	661	42979
161.6	21.739	0.7789	−+	18364	76393	82169	4636591
		0.1704	+-	81	369	803	697851
		0.0254	++	43	135	174	33271
		0.0254	--	146	618	681	42689
162.2	4.348	0.7789	−+	4159	17814	19145	927793
		0.1704	+-	16	62	173	138837
		0.0254	++	10	28	43	6633
		0.0254	--	46	135	141	8463
170.0	26.087	0.7789	−+	63621	264869	270577	5560286
		0.1704	+-	244	957	1447	838233
		0.0254	++	106	451	466	40196
		0.0254	--	508	2215	2282	50979

Table 1: Illustrative example of the numbers of events in each channel for the standard 100 fb⁻¹ 6-point ILC scan with 4 helicity configurations. Columns give the center-of-mass energy, \sqrt{s} , the apportioned integrated luminosity, the fraction for each helicity configuration, $\lambda_{e-}\lambda_{e+}$, and the numbers of events observed in each channel.

$\sqrt{s} = 200$ GeV to 161 GeV. The resulting energy loss function per beam is then fitted using the methods described in [27], resulting in the four standard parameters of the Circe parameterization with values found of 0.70648, 0.25305, 50.507 and -0.7305.

The chosen theoretical model fit parameters are m_W and α_S . For the present studies, α_S , was fixed to 0.12[§].

Other fit parameters related to the normalization and especially the normalization of the signal, include a scale factor for systematic uncertainty on the absolute

[§]Some details on studies related to Γ_W sensitivity were discussed in [10]

No.	Fit Parameter	Comment
1	m_W	Fixed currently to 0.12 0.1% constrained
2	α_S	
3	f_l	
4	ε (lvlv)	Signal efficiency (constrained)
5	ε (qqlv)	
6	ε (qqqq)	
7	σ_B (lvlv)	Background cross-section
8	σ_B (qqlv)	
9	σ_B (qqqq)	
10	A_{LR}^B (lvlv)	Background asymmetry (constrained)
11	A_{LR}^B (qqlv)	
12	A_{LR}^B (qqqq)	
13	β_B (lvlv)	Background shape
14	β_B (qqlv)	
15	β_B (qqqq)	
16	$ P(e^-) $	Assume same for each helicity
17	$ P(e^+) $	Assume same for each helicity
18	σ_Z	Z-like 2-fermion ($f\bar{f}(\gamma)$)
19	A_{LR}^Z	

Table 2: 19-parameter fit for ≥ 4 helicity configuration scans ($-+$, $+-$, $++$, $--$)

Channel	Efficiency (%)	σ_{bkgd}^U (fb)	A_{LR}^B	Eff. syst. (%)	Bkgd syst.	A_{LR}^B syst.
lvlv	87.5	10	0.15	0.1	free	0.025
qqlv	87.5	40	0.30	0.1	free	0.012
qqqq	83.5	200	0.48	0.1	free	0.005

Table 3: Experimental assumptions for the WW event selection near threshold using a polarized scan

integrated luminosity, f_l , and scale factors for corrections to the estimated efficiency in each channel, ε (lvlv), ε (qqlv), ε (qqqq). All four of these uncertainties are constrained within specified uncertainties. The constraint is accomplished by adding a χ^2 penalty contribution.

Parameters 16-19 are used to measure the beam polarization using the Z-like events in a similar manner to [28]. Using equation 1, we see that with 4 cross-section measurements, corresponding to, for example, the $-+$, $+-$, $--$, $++$ helicity configurations, it is possible to measure these four parameters, namely, σ_u , A_{LR} ,

$|P(e^-)|$ and $|P(e^+)|$. This assumes that the polarization magnitudes are identical for positive and negative polarization of the same beam (perfect spin-flip approximation). If this is not the case, or if this assumption needs to be checked it would be possible to take additional data with unpolarized beam or beams to further constrain the polarization model.

The other 9 parameters pertain to the background modeling in each of the 3 channels. The extraction of M_W from the cross-section and its dependence on \sqrt{s} near threshold is sensitive to the understanding of the background. It is expected that the background contribution needs to be determined from data in a robust way. For each channel, the parameters, are the background cross-section, σ_B , the background left-right asymmetry, A_{LR}^B , and a background shape parameter, β_B , allowing for a power-law center-of-mass energy dependence of the background cross-section according to

$$\sigma(\sqrt{s}) = \sigma_B \left(\frac{161}{\sqrt{s}} \right)^{\beta_B}$$

Reasonable guesses for the shape parameter β_B are in the range $[-2, 2]$. This is expected to be a reasonable model for the qqqq channel where background issues are a major concern. Input data-sets use $\beta_B = 0$.

With polarized beams, there is little need to take data at \sqrt{s} values far from threshold to measure the background, but there is a need to measure the polarization and to control the polarization systematics of the background. It is assumed that the background asymmetries can be constrained with background “side-bands”, and these parameters are also implemented with a χ^2 penalty function. The standard fits discussed here with both beams polarized only use the first six background parameters. Other cases of interest are with only electron beam polarization, where one may need to rely more on external measurements of the beam polarization, and some data-taking below threshold. Especially with no polarization, data-taking below threshold would appear mandatory for background control, but it would also be important to understand the \sqrt{s} shape dependence of the background, and the β_B parameters have been introduced to start to address this issue.

Polarized Threshold Scan Study

The improvements include a re-optimization of the fraction of the luminosity associated with each beam helicity configuration which results from the assumed better detector performance. The updated assumptions on the experimental event selection and the associated systematics are given in Table 3. These correspond to a factor of two reduction in the event selection inefficiency and a factor of two reduction in the non-WW backgrounds compared to that essentially achieved with the LEP detectors.

Further improvement beyond these expected performance numbers is not out of the question.

For the current studies A_{LR}^{WW} was set at 1.0. The cross-section for Z-like events was taken to be 150 pb with a value of A_{LR}^Z of 0.19 reflecting a mix of full energy and radiative-return contributions.

Fit parameter	Value	Error
m_W (GeV)	80.388	3.77×10^{-3}
f_l	1.0002	0.924×10^{-3}
ε (lvlv)	1.0004	0.969×10^{-3}
ε (qqlv)	0.99980	0.929×10^{-3}
ε (qqqq)	1.0000	0.942×10^{-3}
σ_B (lvlv) (fb)	10.28	0.92
σ_B (qqlv) (fb)	40.48	2.26
σ_B (qqqq) (fb)	196.37	3.62×10^{-3}
A_{LR}^B (lvlv)	0.15637	0.0247
A_{LR}^B (qqlv)	0.29841	0.0119
A_{LR}^B (qqqq)	0.48012	4.72×10^{-3}
$ P(e^-) $	0.89925	1.27×10^{-3}
$ P(e^+) $	0.60077	9.41×10^{-4}
σ_Z (pb)	149.93	0.052
A_{LR}^Z	0.19062	2.89×10^{-4}

Table 4: Example fit of the 6-point ILC scan with 100 fb^{-1} illustrated in Table 1. In this example, the background β_B shape parameters are fixed to zero, and α_S is fixed at 0.12.

The re-optimized running strategy for 100 fb^{-1} with 90% e^- polarization and 60% e^+ polarization devotes 78% of the integrated luminosity to the “signal” helicity configuration (e_L^-, e_R^+), 17% to the “background” helicity combination (e_R^-, e_L^+) and 5% equally shared amongst the polarization constraining like-sign helicity configurations of (e_L^-, e_L^+) and (e_R^-, e_R^+). The optimization was done assuming 90% electron beam polarization and 60% positron beam polarization. The center-of-mass energies used in the 6-point scan are (160.6, 161.2, 161.4, 161.6, 162.2, 170.0) GeV with integrated luminosities in the ratios of 1:5:5:5:1:6 respectively. The current scan is optimized for measuring M_W . There is room for further optimization and alternative strategies. Alternative scans better suited to measuring Γ_W can also be envisaged.

The results from an ensemble of 1000 toy experiments is found to be an overall uncertainty on M_W of 3.94 MeV. An example of one of these fits is shown in Table 4. In order to assess the effective contribution of the various systematic effects to the overall W mass error, the fits are then re-run for 6 different alternative fits where

the parameters encompassing individual sources of systematic error that are normally fitted for, are fixed to their correct model values. The standard fit and the 6 variations lead to 7 estimates of the W mass uncertainty from which the intrinsic statistical error, the systematics associated with each source of uncertainty, and the total systematic uncertainty are estimated. Note that since all the parameters are fitted for, the overall uncertainty including systematics is statistical in nature and can be improved further with increased integrated luminosity. The results of these 15-parameter fits are shown in Table 5.

Fit type	Uncertainty source	ΔM_W [MeV]	ΔM_W (syst.) [MeV]
fixbkg	Background	3.20	2.30
fixpol	Polarization	3.73	1.27
fixeff	Efficiency	3.86	1.18
fixlum	Luminosity	3.76	0.78
fixALRB	A_{LR}^B	3.86	0.80
fixall	Statistical	2.43	3.10
	Systematic		
standard	Total Error	3.94	

Table 5: Mass errors for various fits for example 100 fb^{-1} 6-point scan with (90%, 60%) beam polarizations

We have also looked into 15-parameter fits with only two scan points such as illustrated in Table 6. The data at $\sqrt{s} = 170 \text{ GeV}$ are quite useful for constraining the normalization parameters of the measurement and is retained. Such fits reach a smaller overall uncertainty. However they are more model-dependent and do little to demonstrate qualitatively the kinematic dependence of the cross-section at threshold. It would seem reasonable to make sure that the data collected would have enough degrees of freedom to test for example the cross-section dependence on Γ_W . The overall results of these scans are reported in Table 7.

There are a number of issues that have not been treated in much depth including theoretical uncertainties, background composition, in particular four-fermion effects, and detailed modeling of event selection performance. However it is thought that the current treatment is appropriate for the current level of study.

\sqrt{s} (GeV)	L (fb ⁻¹)	f	$\lambda_{e^-}\lambda_{e^+}$	N_{ll}	N_{lh}	N_{hh}	N_{RR}
161.4	86.957	0.7111	−+	63443	262469	283058	16927120
161.4	86.957	0.2000	+−	463	1736	3740	3270457
161.4	86.957	0.0444	++	219	922	1023	233371
161.4	86.957	0.0444	−−	997	4043	4463	299399
170.0	13.043	0.7111	−+	29299	121140	123460	2542743
170.0	13.043	0.2000	+−	126	567	900	490497
170.0	13.043	0.0444	++	92	454	404	35300
170.0	13.043	0.0444	−−	445	1905	1927	44740

Table 6: Illustrative example of the numbers of events in each channel for a re-optimized 100 fb⁻¹ 2-point ILC scan with 4 helicity configurations.

$ P(e^-) $ (%)	$ P(e^+) $ (%)	100 fb ⁻¹	500 fb ⁻¹
80	30	6.02	2.88
90	30	5.24	2.60
80	60	4.05	2.21
90	60	3.77	2.12

Table 7: Total W mass uncertainty in MeV from polarized scan near threshold

Summary

A threshold scan with polarized electron and positron beams can yield a precision measurement of M_W at ILC. Errors at the few MeV level can be envisaged. With 100 fb⁻¹, and polarization values of (90%, 60%), the estimated uncertainty is

$$\Delta M_W(\text{MeV}) = 2.4 (\text{stat}) \oplus 3.1 (\text{syst}) \oplus 0.8 (\sqrt{s}) \oplus \text{theory}$$

The ILC design can and should evolve to make this feasible. Positron polarization is extremely helpful in controlling the polarization and background systematics. The highest polarization values would make such a measurement most impactful. Measurements with no positron polarization or even no electron polarization are obviously more challenging.

Eventual experimental precision approaching 2 MeV can be considered at ILC if one is able to dedicate 500 fb⁻¹ to such a measurement, and the physics perspective of the day demands it. Before embarking on such an extended run near threshold, one would certainly want to make sure that the center-of-mass energy systematic is indeed controlled at close to the envisaged level, that the theoretical uncertainties can be controlled adequately, and that such a program offers sufficient complementarity

in the determination of the W mass to data already collected at higher center-of-mass energies synergistic with the main ILC program.

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